CARINGBAH HIGH

2022 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- Reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 - 6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section
- Answer the questions on page 29

Section II – 90 marks (pages 8 - 24)

- Attempt Questions 11–37
- Allow about 2 hours and 45 minutes for this section

Marker's Use Only							
Section I	**Section II**						**Total**
Q1-10	**11 – 16**	**17 – 21**	**22 – 26**	**27 – 30**	**31 – 34**	**35 - 37**	
/10	/14	/16	/14	/16	/14	/16	/100

Section I

10 marks
Attempt Question 1-10
Allow about 15 minutes for this section

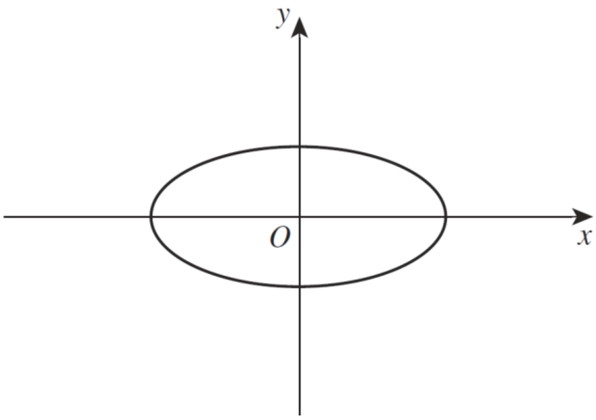
Use the multiple-choice answer sheet for Questions 1-10.

---

1 What is the exact value of $\sec 30° + \tan 30°$?

(A) $\frac{5\sqrt{3}}{6}$ (B) $\frac{3\sqrt{3}}{6}$ (C) $\frac{5\sqrt{3}}{3}$ (D) $\sqrt{3}$

2 Which type of relationship best describes the curve drawn below?



(A) One-to-One (B) One-to-Many

(C) Many-to-One (D) Many-to-Many

3 Matthew recorded the number of runs scored by each member of his cricket team this season. The results were:

25, 85, 96, 104, 110, 122, 124, 129, 144, 144, 160, 205

According to these results:

(A) Both 25 and 205 are outliers

(B) Only 25 is an outlier

(C) Only 205 is an outlier

(D) There are no outliers

4 For $t \neq 0$, find the limiting sum of the geometric series

$$t + \frac{t}{1+t^2} + \frac{t}{\left(1+t^2\right)^2} + \frac{t}{\left(1+t^2\right)^3} + \ldots$$

(A) $\dfrac{1}{1+t^2}$ (B) $\dfrac{t^2}{1+t^2}$ (C) $\dfrac{1+t^2}{t}$ (D) $\dfrac{1+t^2}{t^2}$

5 Joulian records the number of correct questions each student in his class receives in a test that is out of ten

Correct Questions	3	4	5	6	7	8	9	10
Frequency	1	0	3	4	8	4	3	2

Which of the following statements is correct?

(A) Mean = Mode = Median (B) Mean > Mode > Median

(C) Mean > (Mode = Median) (D) Mean < (Mode = Median)

6 Let $f(x)$ be a quadratic polynomial with *x* intercepts at $x = -5$ and $x = 3$.
Which of the following statements is always true?

(A) $f'(-1) = 0$ (B) $f'(0) = 0$

(C) $f'(3) = 0$ (D) $f'(5) = 0$

7 Consider the functions $f(x)=x^2$ and $g(x)=x+2$.

Which of the following expressions is equal to $f(g(x))-g(f(x))$?

(A) $4x+2$ (B) $4x+4$ (C) x^2-x-2 (D) 2

8 What is the domain of the function $f(x)=\ln(x-1)+\sqrt{2-x}$?

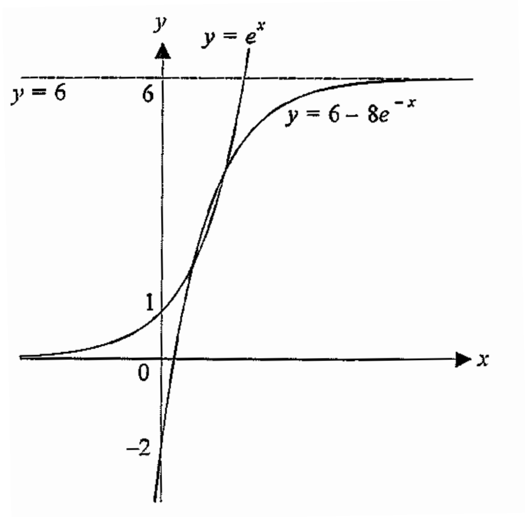
(A) $(1,2)$ (B) $(1,2]$ (C) $[1,2)$ (D) $[1,2]$

9 Which of the following is an expression for $\int \frac{1}{1+\sin x}+\frac{1}{1-\sin x}\,dx$?

(A) $2\sec^2 x+c$ (B) $4\tan x\sec^2 x+c$

(C) $2\tan x+c$ (D) $2x+2\sec x+c$

10



The diagram above shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = e^x$ and $g(x) = 6 - 8e^{-x}$.

The sequence of dilations, reflections and translations that would transform the graph of $y = f(x)$ into the graph of $y = g(x)$ is:

(A) Vertical dilation factor 8, reflect in *x*-axis, reflect in *y*-axis, translate up 6 units

(B) Vertical dilation factor $\frac{1}{8}$, reflect in *x*-axis, reflect in *y*-axis, translate up 6 units

(C) Translate up 6 units, reflect in *x*-axis, reflect in *y*-axis, vertical dilation factor 8

(D) Translate up 6 units, reflect in *x*-axis, reflect in *y*-axis, vertical dilation factor $\frac{1}{8}$

END OF SECTION 1

Marks

Question 11 **2**

Solve $|3x-8|=7$

Question 12 **2**

A circle has the equation $x^2-6x+y^2+4y+k=0$ for some constant *k*.
Find the set of possible values for *k*.

Question 13 **2**

If $\tan x=3$ and $\sin x<0$, find the exact value of $\cos x$.

Question 14 **3**

Marios thinks about three distinct numbers x, $x+4$ and $9x$.
Find all possible sets of these three numbers if they are successive terms in a geometric series.

Question 15

Neve randomly selects two socks from a draw containing thirteen white socks and seven black socks without replacement.

a) Find the probability of selecting a matching pair of colours. **2**

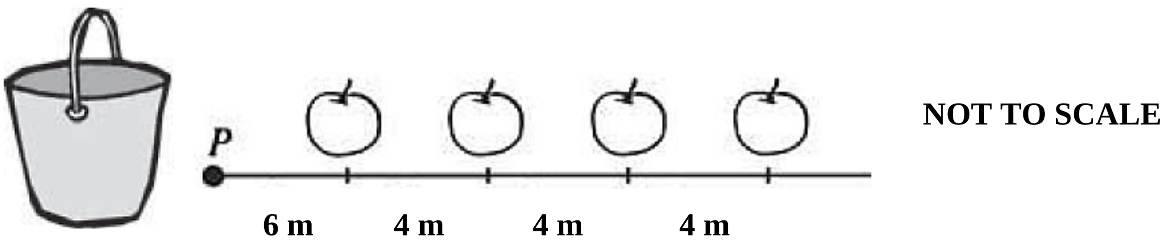
b) Find the probability of **not** selecting a pair of black socks. **1**

Question 16 **2**

Solve $2e^{2x}-e^{x}=0$

Question 17

Three people play a game that involves collecting apples that are placed 4 m apart. The first apple is placed 6 m from point *P*. Players collect the closest apple, then return it to the bucket before collecting the next closest apple and so on.



a) When collecting her last apple, Hannah ran 130 m from point *P* to the apple. How many apples did she collect in total? **1**

b) If Katherine collected 41 apples, how many metres did she run in total? **2**

c) Luke aims to run a marathon whilst playing this game. He achieves this by running a total of 42 432 metres. How many apples did he collect in total? **2**

Question 18 **3**

Solve $2\sin^2 x - 3\sin x - 2 = 0$ for $0 \leq x \leq 2\pi$

Question 19

Differentiate with respect to x:

a) $y = \sin^5 x$ **1**

b) $y = \dfrac{6}{9x-4}$ **1**

Question 20

a) Find $\int \cos 3x \, dx$ **1**

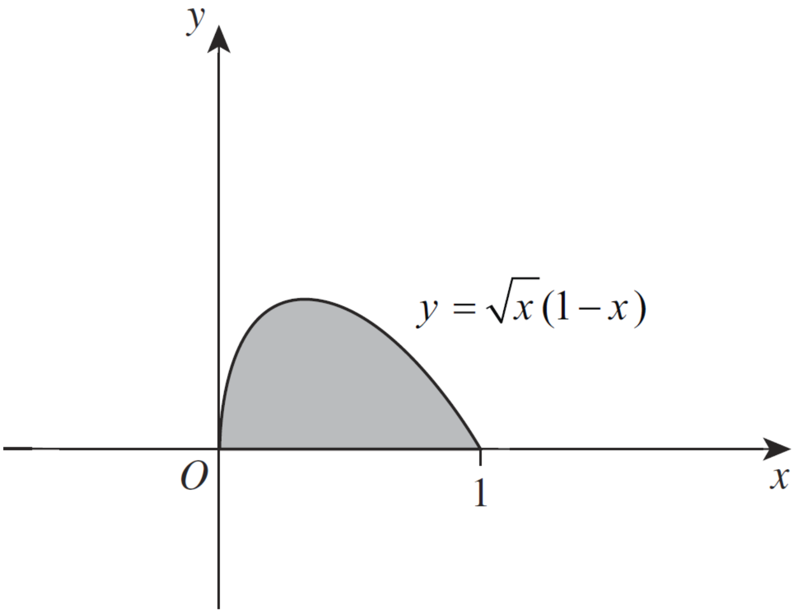
b) Evaluate $\int_0^1 e^{5x-1} dx$ **2**

Question 21 **3**

Prove that $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$

Question 22 **3**

The graph of $y = \sqrt{x}(1 - x)$ is shown below in the domain $0 \le x \le 1$.



Find the area of the shaded region.

Question 23

Maddie owns a company, and has noticed that the profits have been decreasing each year, by 8% of the previous years' profit. In 2013, her company made a profit of $100 000.

a) How much profit will the company make in 2022? **1**

b) In total, how much profit has been made in the ten-year period from 2013 to 2022 inclusive? **1**

c) If this trend continues, what is the total amount of profit the company will make? **1**

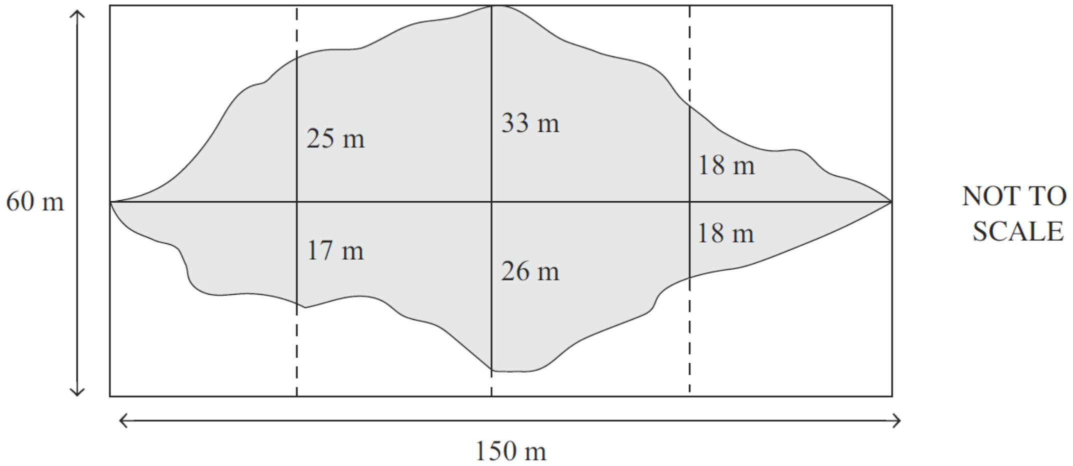
Question 24 **3**

Find the equation of the normal to the curve $y = x\sin x$ at the point where $x = \frac{\pi}{2}$

APPROXIMATELY HALFWAY – 49 marks out of 100 complete at this point

Question 25 **3**

Bowen is an archaeologist and he is excavating a rectangular site with dimensions of 150 metres by 60 metres, as shown below. The site is divided into 8 equal rectangles.



The shaded region indicates the portion of the site that has been excavated.

Using the trapezoidal rule in your working, calculate the percentage of the site that is yet to be excavated, to the nearest whole number.

Question 26

Evaluate $\int_0^2 \frac{6x}{\left(3x^2+1\right)^4}\,dx$ to 3 decimal places **2**

Question 27

Rishi and Jeremy both keep data about the number of goals they score in each game throughout a soccer season. Jeremy places his data into a discrete probability distribution, shown below:

Goals (x)	0	1	2	3	4
$p(x)$	0.35	0.15	0.2	0.25	0.05
$xp(x)$	0	0.15			
$x^2p(x)$	0	0.15			

a) By completing the table above, find the variance of Jeremy's scores **2**

b) Rishi has a standard deviation of 1.4. Determine which player is more consistent and explain. **1**

Question 28 **2**

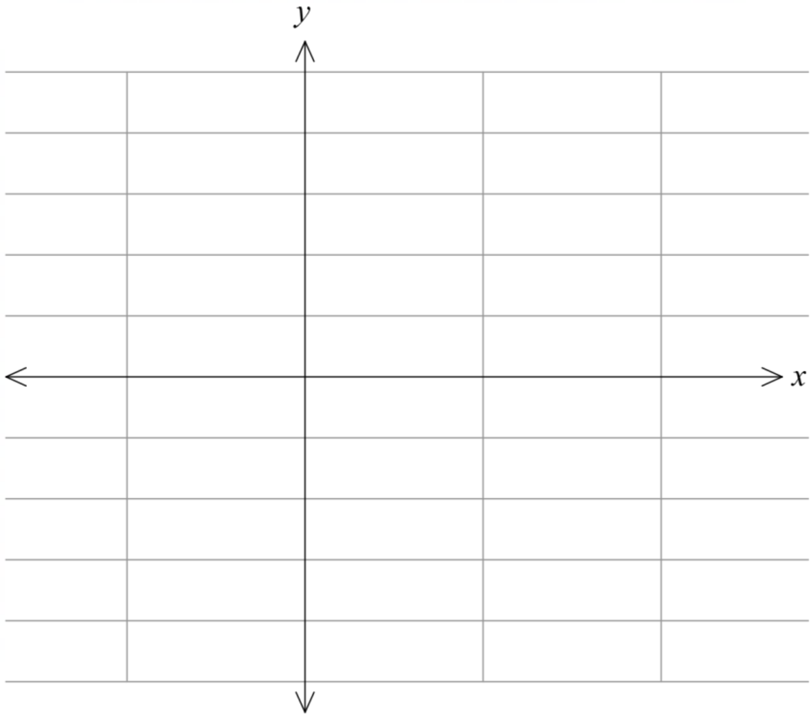
Julie sells lolly bags at a market stall. The bags have a mean weight of 105 g and a standard deviation of 2.5 g. Julie offers a money back refund for any bag under 100 g. If the weights are normally distributed, and she sells 200 lolly bags, how many times will she have to refund customers?

Question 29

Consider the curve $y = 7 + 4x^3 - 3x^4$

a) Find any stationary points and determine their nature. **3**

b) Draw a neat sketch of the curve in the domain $[-1, 2]$ labelling all stationary points **2**



c) Find any values of x such that $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$. **2**

Question 30

Nathan recorded the time people spent training (in hours) and the times they ran (in minutes) in the Sutherland to Surf running race.

Training Hours	**0**	**5**	**7**	**9**	**10**	**12**	**18**	**25**	**32**	**40**	**50**
Race Minutes	**60**	**55**	**65**	**58**	**52**	**50**	**55**	**44**	**48**	**42**	**39**

a) Find the equation of line of best fit (regression line) in the form $y = Bx + A$.
Round the values of A and B to 1 decimal place. **1**

b) Charlie says he spent 22 hours training, what time would this data predict for his race time, correct to the nearest minute? **1**

c) Jackson injured himself in the race, but still completed it, in a time of 80 minutes. Calculate the predicted number of training hours for a time of 80 minutes, then comment on the validity of extrapolating data from these results. **2**

Question 31

The rate at which the depth of water changes in a bay is given by $R = 4\pi \sin \dfrac{\pi t}{6}$ m/h.

a) When Maya goes swimming in the bay, the depth is initially 2 m. **2**
Find an equation for the depth of water, *d* metres, over time *t* hours.

b) Georgia goes swimming 3 hours after Maya. Find the depth of water at this time. **1**

Question 32 **3**

Find the exact area enclosed between the curve $y = \log_e x$, the *y*-axis, and the lines $y = 1$ and $y = 3$.

APPROXIMATELY THREE QUARTERS COMPLETE – 76 marks out of 100 complete

Question 33
Chloe records the lifespan of termites, and determines the that the number of weeks a termite lives can be modelled by the function:

$$f\left(x\right)=\frac{4}{5}-\frac{x^2}{45} \quad \text{for} \quad 3\le x\le 6$$

a) Show that $f\left(x\right)$ is a probability density function. **2**

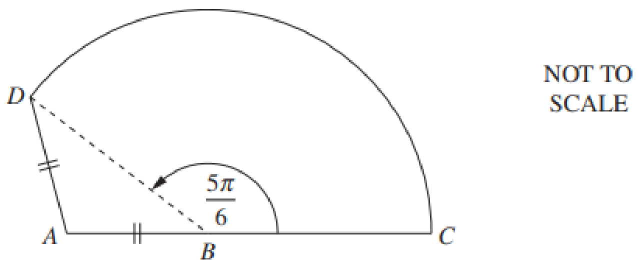
b) Find the CDF (Cumulative Distribution Function). **2**

c) Find the mean lifespan of a termite **1**

Question 34 **3**

Pete worked in a National Park in 1942 and he introduced 18 koalas to the park. By 2002, the number of koalas had grown to 5000. The number of koalas has grown exponentially according to the rule $N = N_0e^{kt}$, where t is the number of years since 1942. N_0 represents the initial amount of koalas and k is a constant. By first finding these values, predict the number of koalas expected to be in the National Park in 2022.

Question 35



Kate has an irregular shaped backyard as seen in the diagram above by shape *ABCD*.
The sector *BCD* has a centre *B*, whilst $AB = AD = 3$ m.

a) Show that the length of *DB* is $3\sqrt{3}$ m. **2**

b) Find the total area of the backyard in exact form. **2**

Question 36

A particle is moving in a straight line. At time t seconds it has a displacement x metres from a fixed point O on the line and velocity $v = 3\sqrt{t+1} - 9$ m/s. Initially the particle is at O.

a) Use integration to show that $x = 2(t+1)\sqrt{t+1} - 9t - 2$. **2**

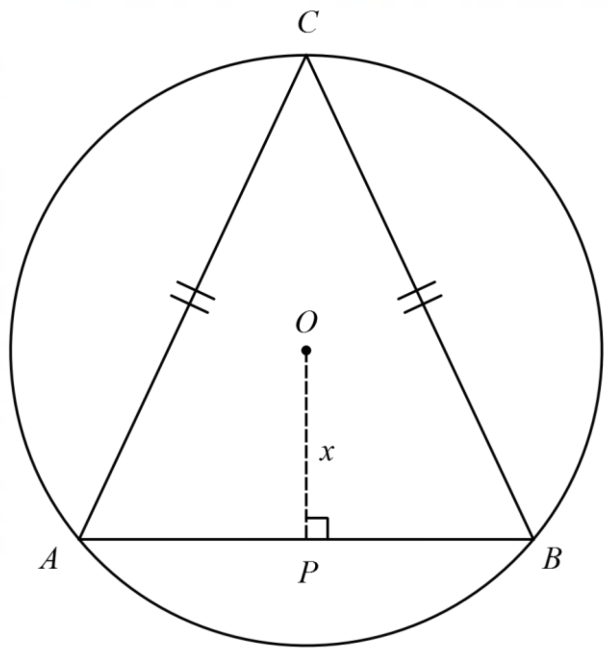
b) Find when the particle is at rest. **1**

c) Find the distance travelled in the first 24 seconds of its motion. **2**

Question 37

An isosceles triangle *ABC*, where $AC = BC$, is inscribed in a circle of radius 10 cm.

$OP = x$ and *OP* bisects *AB* such that $AB \perp OP$.



NOT TO SCALE

a) Show that *A*, the area of triangle *ABC*, is given by $A = (10 + x)\sqrt{100 - x^2}$ **2**

b) Find $\frac{dA}{dx}$, leaving your answer as a single simplified fraction **3**

c) Prove that the triangle with a maximum area is equilateral **2**

END OF EXAM

Solutions

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Mathematics Advanced

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Total marks: 100

Section I – 10 marks (pages 3 - 6)

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Section I	**Section II**						**Total**
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Mark own classes /10	Olivia /14	Kieran /16	Chris /14	Athena /16	Taryn /14	John /16	/100

Section I

10 marks
Attempt Question 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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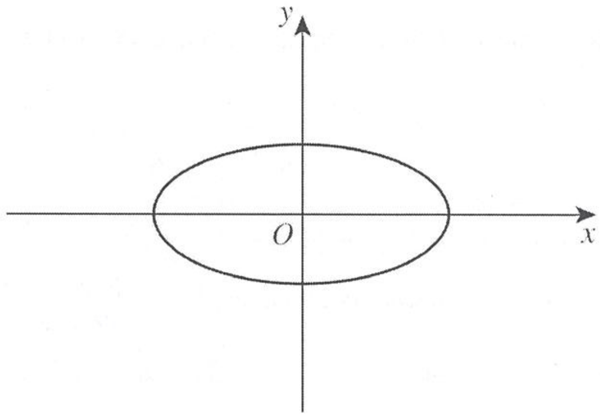
1 What is the exact value of $\sec 30° + \tan 30°$?

(A) $\frac{5\sqrt{3}}{6}$ (B) $\frac{3\sqrt{3}}{6}$ (C) $\frac{5\sqrt{3}}{3}$ (D) $\sqrt{3}$

2 Which type of relationship best describes the curve drawn below?



(A) One-to-One
(B) One-to-Many
(C) Many-to-One
(D) Many-to-Many

3 Matthew recorded the number of runs scored by each member of his cricket team this season. The results were:

25, 85, 96, 104, 110, 122, 124, 129, 144, 144, 160, 205

According to these results:

100 − 1.5 × 44 = 34
144 + 1.5 × 44 = 210

(A) Both 25 and 205 are outliers

(B) Only 25 is an outlier

(C) Only 205 is an outlier

(D) There are no outliers

4 For $t \neq 0$, find the limiting sum of the geometric series

$$t + \frac{t}{1+t^2} + \frac{t}{\left(1+t^2\right)^2} + \frac{t}{\left(1+t^2\right)^3} + \ldots$$

(A) $\frac{1}{1+t^2}$ (B) $\frac{t^2}{1+t^2}$ (C) $\frac{1+t^2}{t}$ (D) $\frac{1+t^2}{t^2}$

$$\frac{t}{1-\frac{1}{1+t^2}} = \frac{t+t^3}{1+t^2-1} = \frac{t(1+t^2)}{t^2} = \frac{1+t^2}{t}$$

5 Joulian records the number of correct questions each student in his class receives in a test that is out of ten

Correct Questions	3	4	5	6	7	8	9	10
Frequency	1	0	3	4	8	4	3	2

Which of the following statements is correct?

Mean 7.08
Mode 7 median 7

(A) Mean = Mode = Median (B) Mean > Mode > Median

(C) Mean > (Mode = Median) (D) Mean < (Mode = Median)

6 Let $f(x)$ be a quadratic polynomial with x intercepts at $x = -5$ and $x = 3$. Which of the following statements is always true?

(A) $f'(-1) = 0$ (B) $f'(0) = 0$

(C) $f'(3) = 0$ (D) $f'(5) = 0$

7 Consider the functions $f(x)=x^2$ and $g(x)=x+2$.

Which of the following expressions is equal to $f(g(x))-g(f(x))$?

(A) $4x+2$ (B) $4x+4$ (C) x^2-x-2 (D) 2

$(x+2)^2-(x^2+2)$

$= x^2+4x+4-x^2-2$

$= 4x+2$.

8 What is the domain of the function $f(x)=\ln(x-1)+\sqrt{2-x}$?

$x>1$

$x\leq 2$

(A) $(1,2)$ (B) $(1,2]$ (C) $[1,2)$ (D) $[1,2]$

9 Which of the following is an expression for $\int \frac{1}{1+\sin x}+\frac{1}{1-\sin x}\,dx$?

(A) $2\sec^2 x+c$

(B) $4\tan x\sec^2 x+c$

(C) $2\tan x+c$

(D) $2x+2\sec x+c$

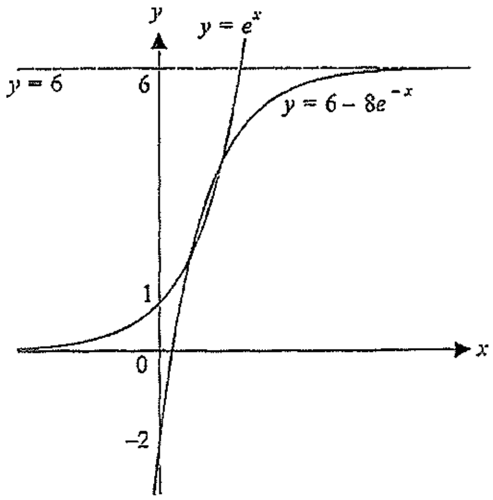
$\int \frac{1-\sin x+1+\sin x}{1-\sin^2 x}$

$= \int \frac{2}{\cos^2 x}$

$= \int 2\sec^2 x$

$= 2\tan x+c$

10



The diagram above shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = e^x$ and $g(x) = 6 - 8e^{-x}$.

The sequence of dilations, reflections and translations that would transform the graph of $y = f(x)$ into the graph of $y = g(x)$ is:

(A) Vertical dilation factor 8, reflect in x-axis, reflect in y-axis, translate up 6 units

(B) Vertical dilation factor $\frac{1}{8}$, reflect in x-axis, reflect in y-axis, translate up 6 units

(C) Translate up 6 units, reflect in x-axis, reflect in y-axis, vertical dilation factor 8

(D) Translate up 6 units, reflect in x-axis, reflect in y-axis, vertical dilation factor $\frac{1}{8}$

END OF SECTION 1

Marks

Question 11 **2**

Solve $|3x-8|=7$

$3x - 8 = 7$ $\quad\quad$ $3x - 8 = -7$

$3x = 15$ $\quad\quad$ $3x = 1$

$x = \frac{15}{3}$ $\quad\quad$ $x = \frac{1}{3}$ (1)

$x = 5$ (1)

Question 12 **2**

A circle has the equation $x^2 - 6x + y^2 + 4y + k = 0$ for some constant k.
Find the set of possible values for k.

$(x-3)^2 + (y+2)^2 = -k + 9 + 4$

$(x-3)^2 + (y+2)^2 = 13 - k$ (1)

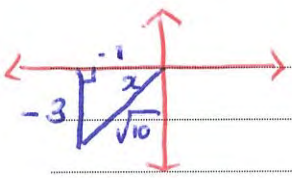
$\therefore 13 - k > 0$

$13 > k$

$\therefore k < 13$ (1)

Question 13 **2**

If $\tan x = 3$ and $\sin x < 0$, find the exact value of $\cos x$.



$\cos x = \frac{-1}{\sqrt{10}}$

Question 14 3

Marios thinks about three distinct numbers x, $x+4$ and $9x$.
Find all possible sets of these three numbers if they are successive terms in a geometric series.

$\frac{9x}{x+4} = \frac{x+4}{x}$ (1)

$9x^2 = x^2 + 8x + 16$

$8x^2 - 8x - 16 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$ (1)

If $x = 2$, the set is $2, 6, 18$

If $x = -1$, the set is $-1, 3, -9$ (1)

Question 15

Neve randomly selects two socks from a draw containing thirteen white socks and seven black socks without replacement.

a) Find the probability of selecting a matching pair of colours. **2**

$\left(\frac{13}{20} \times \frac{12}{19}\right) + \left(\frac{7}{20} \times \frac{6}{19}\right) = \frac{99}{190}$

(1) (1)

b) Find the probability of **not** selecting a pair of black socks. **1**

$1 - \left(\frac{7}{20} \times \frac{6}{19}\right) = \frac{169}{190}$

Question 16 2

Solve $2e^{2x} - e^{x} = 0$

$e^x(2e^x - 1) = 0$

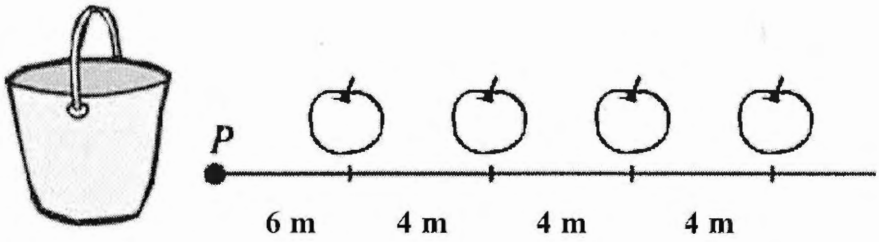
$e^x \neq 0 \quad \therefore 2e^x = 1$

$e^x = 1/2$ (1)

$x = \log_e\left(\frac{1}{2}\right)$ (1) or $x = -\log_e 2$

Question 17

Three people play a game that involves collecting apples that are placed 4 m apart. The first apple is placed 6 m from point P. Players collect the closest apple, then return it to the bucket before collecting the next closest apple and so on.



NOT TO SCALE

a) When collecting her last apple, Hannah ran 130 m from point P to the apple. How many apples did she collect in total? **1**

$a = 6$, $d = 4$

$130 = 6 + (n-1)4$

$124 = 4n - 4$

$128 = 4n$

$n = 32$ ∴ she collected 32 apples

b) If Katherine collected 41 apples, how many metres did she run in total? **2**

$a = 12$, $d = 8$, $n = 41$

$S_{41} = \frac{41}{2}[24 + 8(41-1)]$ (1)

$= 41/2\,[24 + 320]$

$= 7052\text{m}$ (1)

c) Luke aims to run a marathon whilst playing this game. He achieves this by running a total of 42 432 metres. How many apples did he collect in total? **2**

$42432 = \frac{n}{2}[24 + 8(n-1)]$

$84864 = 24n + 8n^2 - 8n$

$0 = 8n^2 + 16n - 84864$

$0 = n^2 + 2n - 10608$ (1)

$0 = (n+104)(n-102)$

$n = -104$ or 102

$n \neq 104$ ∴ Luke collected 102 apples (1)

Question 18 3

Solve $2\sin^2 x - 3\sin x - 2 = 0$ for $0 \le x \le 2\pi$

let $\sin x = u$

$2u^2 - 3u - 2 = 0$ ① $x -4$ | -4 ; $+ -3$ | 1

$2u^2 - 4u + u - 2 = 0$

$2u(u-2) + (u-2) = 0$

$(u-2)(2u+1) = 0$

$u = 2$ or $u = -\frac{1}{2}$ ①

$\sin x \neq 2 \therefore \sin x = -\frac{1}{2}$

$x = 210°, 330°$

$= \frac{7\pi}{6}, \frac{11\pi}{6}$ ①

Question 19

Differentiate with respect to x:

a) $y = \sin^5 x$ 1

$y = (\sin x)^5$

$\frac{dy}{dx} = 5(\sin x)^4 \cdot \cos x$

$= 5\sin^4 x \cos x$

b) $y = \dfrac{6}{9x-4}$ 1

$y = 6(9x-4)^{-1}$

$\frac{dy}{dx} = -6(9x-4)^{-2} \times 9$

$= \dfrac{-54}{(9x-4)^2}$

Question 20

a) Find $\int \cos 3x \, dx$ **1**

$\frac{1}{3}\sin 3x + C$

b) Evaluate $\int_0^1 e^{5x-1} dx$ **2**

$\frac{1}{5}\left[e^{5x-1}\right]_0^1$ (1)

$= \frac{1}{5}\left[e^4 - \frac{1}{e}\right]$

$= \frac{e^4}{5} - \frac{1}{5e}$

$= \frac{e^5 - 1}{5e}$ (1)

Question 21 **3**

Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

LHS $\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} + \frac{(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$ (1)

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$ (1)

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

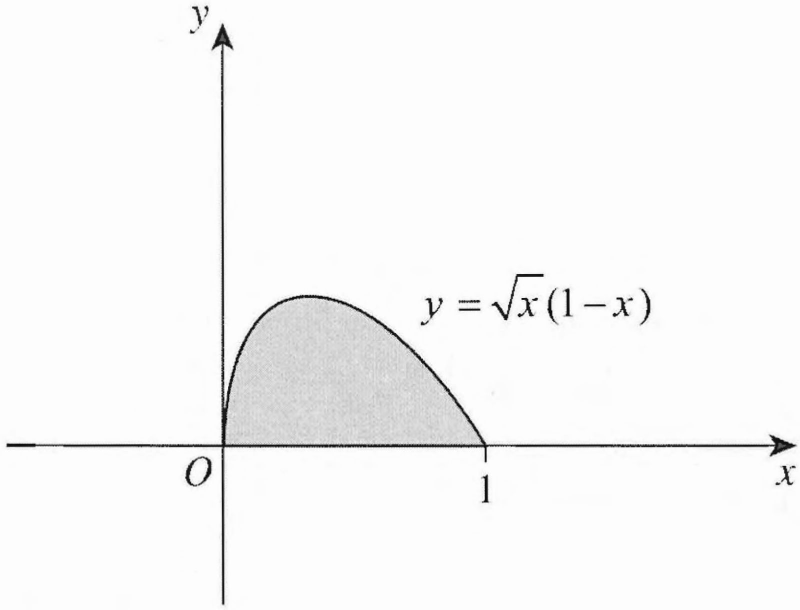
$= \frac{2}{\sin\theta}$

$= 2\operatorname{cosec}\theta$ (1)

$=$ RHS

Question 22 **3**

The graph of $y = \sqrt{x}(1-x)$ is shown below in the domain $0 \le x \le 1$.



Find the area of the shaded region.

$A = \int_0^1 \sqrt{x}(1-x)\,dx$

$= \int_0^1 (\sqrt{x} - x\sqrt{x})\,dx$

$= \int_0^1 \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$ ①

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2}\right]_0^1$

$= \left[\frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5}\right]_0^1$ ①

$= \frac{2}{3} - \frac{2}{5}$

$= \frac{4}{15}u^2$ ①

Question 23

Maddie owns a company, and has noticed that the profits have been decreasing each year, by 8% of the previous years' profit. In 2013, her company made a profit of \$100 000.

a) How much profit will the company make in 2022? **1**

$100000 + 0.92 \times 100000 + 0.92^2 \times 100000 \ldots$

$a = 100000, \; r = 0.92$

$T_{10} = 100000(0.92)^9$

$= \$47216.14$

b) In total, how much profit has been made in the ten-year period from 2013 to 2022 inclusive? **1**

$S_{10} = \dfrac{100000(1-0.92^{10})}{1-0.92}$

$= \$707014.43$

c) If this trend continues, what is the total amount of profit the company will make? **1**

$S_\infty = \dfrac{100000}{1-0.92}$

$= \$1250000$

Question 24 **3**

Find the equation of the normal to the curve $y = x\sin x$ at the point where $x = \dfrac{\pi}{2}$

$y' = \sin x + x\cos x$ (1)

at $x = \frac{\pi}{2}$

$m_T = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2}$

$= 1$

$\therefore m_N = -1$ (1)

at $x = \frac{\pi}{2}$ $y = \frac{\pi}{2}\sin\frac{\pi}{2}$

$= \frac{\pi}{2}$

$\rightarrow y - \frac{\pi}{2} = -1(x - \pi/2)$

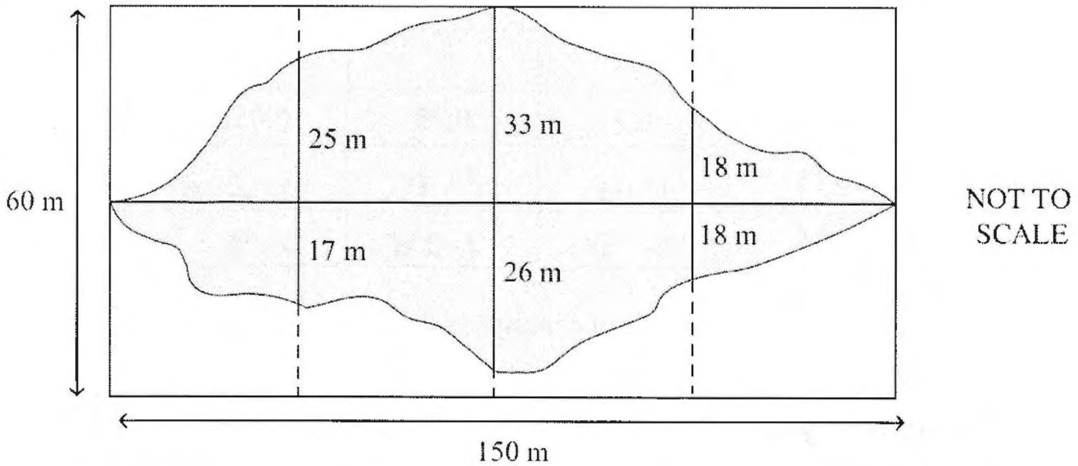
$y - \pi/2 = -x + \frac{\pi}{2}$

$y = -x + \pi$ (1)

APPROXIMATELY HALFWAY – 49 marks out of 100 complete at this point

Question 25 3

Bowen is an archaeologist and he is excavating a rectangular site with dimensions of 150 metres by 60 metres, as shown below. The site is divided into 8 equal reetangles.



The shaded region indicates the portion of the site that has been excavated.

Using the trapezoidal rule in your working, calculate the percentage of the site that is yet to be excavated, to the nearest whole number.

$= \frac{37.5}{2}(0 + 0 + 2(42 + 59 + 36))$

$= 5137.5$ (excavated) ①

$60 \times 150 = 9000$ (total)

$9000 - 5137.5 = 3862.5$ (yet to be excavated) ①

$\frac{3862.5}{9000} \times 100 = 43\%$ ①

Question 26

Evaluate $\int_0^2 \frac{6x}{(3x^2+1)^4} dx$ to 3 decimal places 2

$\int_0^2 6x(3x^2+1)^{-4} dx$

$= \left[\frac{(3x^2+1)^{-3}}{-3}\right]_0^2$ ①

$= \frac{13^{-3}}{-3} - \frac{1^{-3}}{-3}$

$= 0.333$ ①

Question 27

Rishi and Jeremy both keep data about the number of goals they score in each game throughout a soccer season. Jeremy places his data into a discrete probability distribution, shown below:

Goals (x)	0	1	2	3	4
$p(x)$	0.35	0.15	0.2	0.25	0.05
$xp(x)$	0	0.15	0.4	0.75	0.2
$x^2p(x)$	0	0.15	0.8	2.25	0.8

a) By completing the table above, find the variance of Jeremy's scores **2**

$\mu = 1.5$ ①

$VAR(x) = \Sigma x^2 p(x) - \mu^2$

$= 4 - 1.5^2$

$= 1.75$ ①

b) Rishi has a standard deviation of 1.4. Determine which player is more consistent and explain. **1**

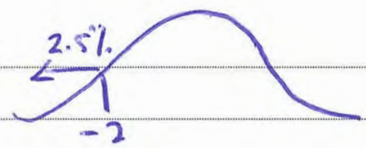
Rishi: $\sigma = 1.4$ Jeremy has lower standard

Jeremy $\sigma = 1.32$ deviation, ∴ is more consistent

Question 28 **2**

Julie sells lolly bags at a market stall. The bags have a mean weight of 105 g and a standard deviation of 2.5 g. Julie offers a money back refund for any bag under 100 g. If the weights are normally distributed, and she sells 200 lolly bags, how many times will she have to refund customers?

z-score −2



① for either 2.5% or z-score −2

$2.5\% \times 200 = 5$ times.

①

Question 29

Consider the curve $y = 7 + 4x^3 - 3x^4$

a) Find any stationary points and determine their nature. **3**

(1) for correct St. Pts
(1) for correct nature
(1) ~~for testing concavity~~

$y' = 12x^2 - 12x^3$

$12x^2(1-x) = 0$

$\therefore$ Stationary Pt at $x=0$ and $x=1$

$y=7$ $\quad y=8$

$y'' = 24x - 36x^2$

at $x=0$ $\quad y''=0$ $\quad \therefore (0,7)$ possible POI.

at $x=1$ $\quad y''=-12$ $\quad \therefore (1,8)$ is concave down.

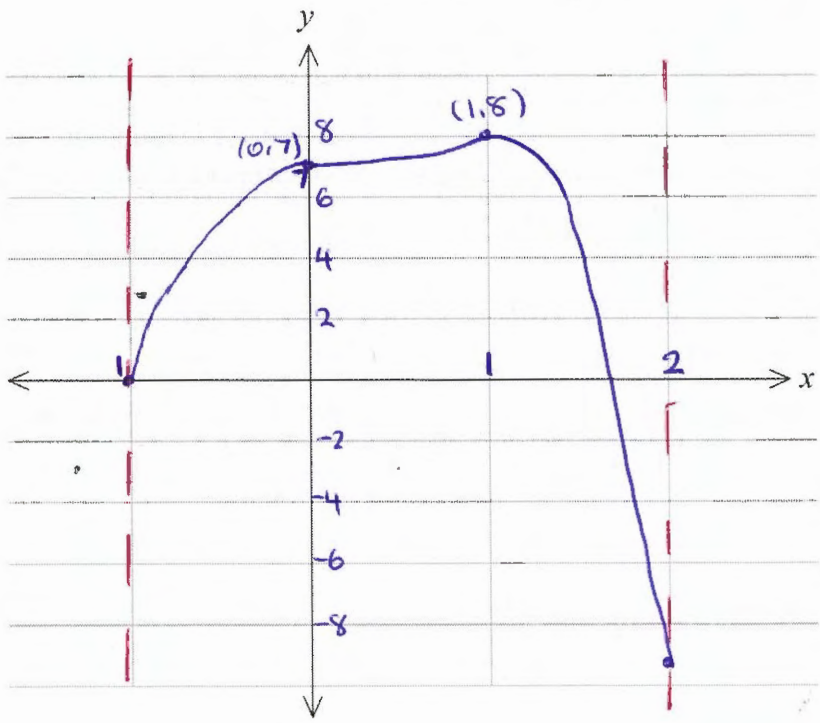
x	-1	0	$\frac{1}{2}$
y''	-60	0	3

Concavity changes

$\therefore (0,7)$ is a horizontal point of inflexion.

$(1,8)$ is a maximum TP.

b) Draw a neat sketch of the curve in the domain $[-1, 2]$ labelling all stationary points **2**



c) Find any values of x such that $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$. **2**

for $y'' < 0$, $24x - 36x^2 < 0$

$12x(2-3x) < 0$

$x < 0$ or $x > \frac{2}{3}$

for $y' > 0$

$x < 1$, $x \neq 0$

$\therefore x < 0$ and $\frac{2}{3} < x < 1$

(1) (1)

Question 30

Nathan recorded the time people spent training (in hours) and the times they ran (in minutes) in the Sutherland to Surf running race.

Training Hours	0	5	7	9	10	12	18	25	32	40	50
Race Minutes	60	55	65	58	52	50	55	44	48	42	39

a) Find the equation of line of best fit (regression line) in the form $y = Bx + A$. Round the values of A and B to 1 decimal place. **1**

$y = -0.4x + 59.9$

b) Charlie says he spent 22 hours training, what time would this data predict for his race time, correct to the nearest minute? **1**

$y = -0.4 \times 22 + 59.9$

$= 51$ mins

c) Jackson injured himself in the race, but still completed it, in a time of 80 minutes. Calculate the predicted number of training hours for a time of 80 minutes, then comment on the validity of extrapolating data from these results. **2**

$80 = -0.4x + 59.9$

$20.1 = -0.4x$

$x = -50.25$ ①

Negative training hours is not possible, suggests extrapolating data is not valid. ①

Question 31

The rate at which the depth of water changes in a bay is given by $R = 4\pi \sin\frac{\pi t}{6}$ m/h.

a) When Maya goes swimming in the bay, the depth is initially 2 m. **2**
Find an equation for the depth of water, d metres, over time t hours.

$d = \int 4\pi \sin\frac{\pi t}{6}\, dt$

$d = \frac{-24\pi}{\pi}\cos\frac{\pi t}{6} + C$

$d = -24\cos\frac{\pi t}{6} + C$ ①

at $t = 0$, $d = 2$

$2 = -24\cos 0 + c$

$2 = -24 + c$

$c = 26$ $\therefore d = -24\cos\frac{\pi t}{6} + 26$ ①

b) Georgia goes swimming 3 hours after Maya. Find the depth of water at this time. **1**

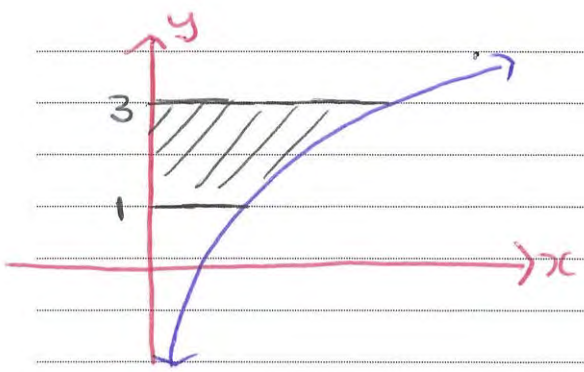
at $t = 3$

$d = -24\cos\frac{\pi}{2} + 26$

$d = 26$ m

Question 32 **3**

Find the exact area enclosed between the curve $y = \log_e x$, the y-axis, and the lines $y = 1$ and $y = 3$.



$y = \log_e x$

$\therefore x = e^y$ ①

$\text{Area} = \int_1^3 e^y\, dy$

$= [e^y]_1^3$ ①

$= (e^3 - e)\, u^2$

or $e(e^2 - 1)\, u^2$ ①

APPROXIMATELY THREE QUARTERS COMPLETE – 76 marks out of 100 complete

Question 33

Chloe records the lifespan of termites, and determines the that the number of weeks a termite lives can be modelled by the function:

$$f(x)=\frac{4}{5}-\frac{x^2}{45} \quad \text{for} \quad 3 \le x \le 6$$

a) Show that $f(x)$ is a probability density function. **2**

$\int_3^6 \frac{4}{5}-\frac{x^2}{45}\,dx$

$=\left[\frac{4x}{5}-\frac{x^3}{135}\right]_3^6$ (1)

$=\left(\frac{24}{5}-\frac{216}{135}\right)-\left(\frac{12}{5}-\frac{27}{135}\right)$

$=\frac{16}{5}-\frac{11}{5}=1$ (1) $\therefore$ PDF

b) Find the CDF (Cumulative Distribution Function). **2**

$\int_3^x \left(\frac{4}{5}-\frac{t^2}{45}\right)dt$ (1)

$=\left[\frac{4t}{5}-\frac{t^3}{135}\right]_3^x$

$=\left(\frac{4x}{5}-\frac{x^3}{135}\right)-\frac{11}{5}$

$=\frac{108x-x^3-297}{135}$ (1)

c) Find the mean lifespan of a termite **1**

$\mu=\int_3^6 x f(x)$

$=\left[\frac{4x^2}{10}-\frac{x^4}{180}\right]_3^6$

$=\left[\left(\frac{144}{10}-\frac{1296}{180}\right)-\left(\frac{36}{10}-\frac{81}{180}\right)\right]$

$=4.05$ days

Question 34 3

Pete worked in a National Park in 1942 and he introduced 18 koalas to the park. By 2002, the number of koalas had grown to 5000. The number of koalas has grown exponentially according to the rule $N = N_0 e^{kt}$, where t is the number of years since 1942. N_0 represents the initial amount of koalas and k is a constant. By first finding these values, predict the number of koalas expected to be in the National Park in 2022.

$N_0 = 18$

$5000 = 18e^{60k}$ (1)

$\frac{5000}{18} = e^{60k}$

$60k = \log_e\left(\frac{5000}{18}\right)$

$k = 0.0937....$ (1)

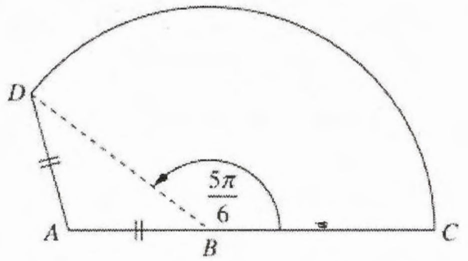
$N = 18e^{0.0937..t}$

in 2022, $t = 80$

$N = 18e^{0.0937..\times 80}$

$N = 32624$ koalas (1)

Question 35



NOT TO SCALE

Kate has an irregular shaped backyard as seen in the diagram above by shape *ABCD*. The sector *BCD* has a centre *B*, whilst $AB = AD = 3$ m.

a) Show that the length of *DB* is $3\sqrt{3}$ m. 2

$\angle ABD = \pi/6$ $\angle ADB = \pi/6$

$\therefore \angle DAB = 2\pi/3$

$DB^2 = 3^2+3^2-2\times 3^2 \times \cos\frac{2\pi}{3}$ (1)

$DB^2 = 27$ $\therefore DB = \sqrt{27} = 3\sqrt{3}$ (1)

b) Find the total area of the backyard in exact form. 2

$\Delta = \frac{1}{2}\times 3^2 \times \sin\frac{2\pi}{3}$

$= \frac{9}{2}\times\frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}$ (1)

Sector $= \frac{1}{2}\times(3\sqrt{3})^2\times\frac{5\pi}{6}$

$= \frac{135\pi}{12} = \frac{45\pi}{4}$

$\therefore$ total $= \frac{9\sqrt{3}+45\pi}{4}$ m^2 (1)

Question 36

A particle is moving in a straight line. At time t seconds it has a displacement x metres from a fixed point O on the line and velocity $v = 3\sqrt{t+1} - 9$ m/s. Initially the particle is at O.

a) Use integration to show that $x = 2(t+1)\sqrt{t+1} - 9t - 2$. **2**

$x = \int (3(t+1)^{1/2} - 9)\,dt$

$= \frac{3(t+1)^{3/2}}{3/2} - 9t + C$

$= 2(t+1)^{3/2} - 9t + C$ (1)

at $t=0$, $x=0$

$0 = 2 - 0 + C$

$C = -2$ (1)

$\therefore x = 2(t+1)\sqrt{t+1} - 9t - 2$

b) Find when the particle is at rest. **1**

rest $v = 0$

$3\sqrt{t+1} = 9$

$\sqrt{t+1} = 3$

$t+1 = 9$

$t = 8$ secs

c) Find the distance travelled in the first 24 seconds of its motion. **2**

at 8 sec

$x = 2(9)(3) - 72 - 2 = -20$ (1)

at $t = 24$ $x = 2(25)(5) - 216 - 2 = 32$

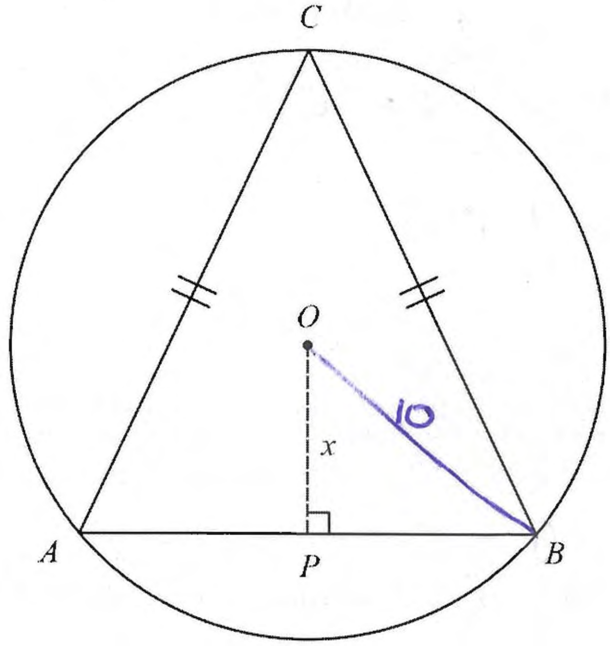
total $= 20 + 20 + 32$

$= 72$ m (1)

Question 37

An isosceles triangle ABC, where $AC = BC$, is inscribed in a circle of radius 10 cm.

$OP = x$ and OP bisects AB such that $AB \perp OP$.



NOT TO SCALE

a) Show that A, the area of triangle ABC, is given by $A = (10 + x)\sqrt{100 - x^2}$ **2**

$PB^2 = 10^2 - x^2$

$PB = \sqrt{100 - x^2}$

$\therefore AB = 2\sqrt{100 - x^2}$ (1)

Δ height $= x + 10$

$\therefore$ Area $= \frac{1}{2} \times 2\sqrt{100 - x^2} \times (x + 10)$ (1)

$A = (10 + x)\sqrt{100 - x^2}$ (as required)

b) Find $\frac{dA}{dx}$, leaving your answer as a single simplified fraction **3**

$$A = (10+x)(100-x^2)^{1/2}$$

$$\frac{dA}{dx} = (10+x)\times\frac{1}{2}(100-x^2)^{-\frac{1}{2}}\times -2x + (100-x^2)^{\frac{1}{2}} \quad (1)$$

$$= \frac{-x(10+x)}{(100-x^2)^{1/2}} + \frac{(100-x^2)^{1/2}\times(100-x^2)^{1/2}}{(100-x^2)^{1/2}}$$

$$= \frac{-x(10+x)+(100-x^2)}{(100-x^2)^{1/2}} \quad (1)$$

$$= \frac{-10x-x^2+100-x^2}{(100-x^2)^{1/2}}$$

$$= \frac{-2x^2-10x+100}{(100-x^2)^{1/2}}$$

$$= \frac{-2(x^2+5x-50)}{\sqrt{100-x^2}} \quad (1)$$

c) Prove that the triangle with a maximum area is equilateral **2**

For maximum $\frac{dA}{dx}=0$

$\therefore -2(x^2+5x-50)=0$

$-2(x+10)(x-5)=0$

$\therefore x=-10$ or 5

$x\neq -10 \quad \therefore x=5$

x	4	5	6
y'	3.1	0	-4

↗ ↘

$\therefore x=5$ is a max. (1)

If $x=5$, $AB = 2\sqrt{75}$ and $AP = 5\sqrt{3}$.

$= 10\sqrt{3}$

$AC^2 = (5\sqrt{3})^2 + 15^2$

$AC^2 = 300$ (1)

$AC = \sqrt{300} = 10\sqrt{3}$ $\quad\therefore AC = CB = AB$ making an equilateral triangle.

END OF EXAM